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Vortex characteristics in Fourier and non-Fourier heat conduction based on heat flux rotation

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ABSTRACT

Vortex is one of the most important characteristics for flow field, but for heat conduction, vortex problems have not been studied. In this paper, heat vortex is mainly investigated from the viewpoint of heat flux rotation. The heat flux rotation is calculated and discussed for Fourier's law and typical non-Fourier heat conduction models, including the Cattaneo-Vernotte (CV) model, Jeffrey model, Guyer-Krumhansl (GK) model and dual-phase-lagging (DPL) model. It is found that the change rule of heat flux rotation is only determined by the heat conduction law, and the change rule can also reflect certain relaxation process in non-Fourier heat conduction. These conclusions can provide a perspective for proving non-Fourier heat conduction, which is more rigorous than heat wave phenomena or finite propagation velocity. The heat flux rotation can also shows particular characteristics of non-Fourier heat conduction, which is quite different from mechanical phenomena. Different from the heat flux rotation, the entropy flux rotation for non-Fourier heat conduction does not change in an established rule.

1. Introduction

Vortex is one of the most important and fundamental definition for flow field. In fluid mechanics, fairly in-depth discussions have been made on vortex, which is called as "the sinews and muscles of fluid motions" [1], showing the indispensability of vortex. Vortex motion is closely related to shear in fluid flow, which reflects the constitutive relations and flow laws in fluid mechanics. In many fluid mechanics problems, fluid flow is often coupled with heat transfer, which relates to other flow fields about heat conduction, including heat flow field and entropy flow field. Besides fluid mechanics, these fields are rather important for heat transfer itself and irreversible thermodynamics. For heat flow and entropy flow, vortex motion is also a necessary movement mode, and similar to fluid flow, vortex in heat flow field or entropy flow field can also reflect characteristics of heat transfer laws. However, vortex is not much investigated from the viewpoint of heat transfer laws.

Different heat conduction models have been proposed, and Fourier's law of heat conduction is most frequently used. Fourier's law shows the connection between temperature gradient and heat flux

$$q + \lambda_F \nabla T = 0, \tag{1}$$

where q is the heat flux, T is the temperature and λ_F is the Fourier thermal conductivity, which is generally expressed as $\lambda_F = \lambda_F(T)$ (including constant). The energy conservation equation is

$$\phi - \nabla \cdot q = \rho c_V \frac{\partial T}{\partial t}, \tag{2}$$

where ϕ is the heat source, ρ is the mass density and c_V is the specific heat. Eqs. (1) and (2) can be combined to give the heat conduction equation

$$\nabla[\lambda_F(T)\nabla T] + \phi = \rho c_V \frac{\partial T}{\partial t}. \tag{3}$$

In recent years, it is revealed that Fourier's law cannot predict the supertransient and high heat flux processes [2–7]. To get over these limitations, several modified non-Fourier heat conduction models were proposed. The Cattaneo-Vernotte (CV) model [8,9] is the most typical and classical one

$$q + \tau \frac{\partial q}{\partial t} + \lambda_{CV} \nabla T = 0, \tag{4}$$

where τ is the thermal relaxation time and λ_{CV} is the thermal conductivity for the CV model. Jeffrey model [3] is an extension of the CV model, which could be considered as a linear superposition of Fourier's law and the CV model

$$q + \tau \frac{\partial q}{\partial t} + \lambda(T)\nabla T + \lambda_F(T)\tau \frac{\partial}{\partial t}(\nabla T) = 0, \tag{5}$$

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where $\lambda(T)$ is the total thermal conductivity. The Guyer-Krumhansl (GK) model [10] is a classical model for pure phonon heat conduction

$$\frac{\partial q}{\partial t}+\frac{c^2 c_l}{3}\nabla T+\frac{1}{\tau_R}q=\frac{3\tau_N c^2}{5}\nabla(\nabla\cdot q), \tag{6}$$

where c_l is the phonon specific heat, τ_N is the single-phonon relaxation time for normal processes, τ_R is the momentum loss relaxation time and c is the isothermal first-sound velocity. The dual-phase-lagging (DPL) model [11], which considers the influence of temperature relaxation, is often Taylor expanded as [12–15]

$$q+\tau_q\frac{\partial q}{\partial t}+\lambda_{DPL}\left[\nabla T+\tau_T\frac{\partial}{\partial t}(\nabla T)\right]=0, \tag{7}$$

where λ_{DPL} is the thermal conductivity for the CV model, τ_q is the relaxation time of the heat flux and τ_T is the relaxation time of temperature gradient. There are also other heat conduction models, such as the two-temperature (T-T) model [16], the thermomass theory for heat conduction [17–21], alternative approaches to the analysis of the diffusion equation [22–24] and some extensions of these models [25–29].

There is no uniform mathematical definition of vortex at present. In fluid mechanics, there are different definitions for vortex [30–33], including rotation [30], Q-criterion [31], Δ-criterion [32], λ_2-criterion [33], etc. To simplify the problem, we only use the rotation to discuss vortex in heat conduction. In this paper, the heat flux rotation is calculated and discussed for Fourier's law and several typical non-Fourier heat conduction models, including the CV model, Jeffrey model, the GK model and the DPL model. These discussion could be applied to prove non-Fourier heat conduction. The heat flux rotation can also reflect particular characteristics of non-Fourier heat conduction, which is quite different from mechanical phenomenon. As another common and important flux in heat conduction processes, the entropy flux rotation is also calculated for different models.

2. Heat flux rotation

For Fourier's law, the heat flux rotation is

$$(\nabla\times q)_F=-\nabla\times[\lambda_F(T)\nabla T]=-\nabla\times\nabla\left(\int_0^T\lambda_F(\varepsilon)d\varepsilon\right)=0, \tag{8}$$

which shows that for Fourier's law, the heat flux field must be non-rotating. For the CV model, taking rotation [34] of Eq. (4) leads to

$$\nabla\times q+\tau\frac{\partial(\nabla\times q)}{\partial t}+\nabla\times[\lambda_{CV}(T)\nabla T]=0. \tag{9}$$

Then we can obtain [34]

$$(\nabla\times q)_{CV}+\tau\frac{\partial(\nabla\times q)_{CV}}{\partial t}=0, \tag{10}$$

whose solution is

$$(\nabla\times q)_{CV}=(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau}}. \tag{11}$$

Eq. (11) shows the exponential decay of the heat flux rotation, and the decay rate is determined by the relaxation time. The rotation [34] of Jeffrey model Eq. (5) is

$$(\nabla\times q)+\tau\frac{\partial(\nabla\times q)}{\partial t}+\nabla\times\lambda(T)\nabla T+\tau\frac{\partial}{\partial t}[\nabla\times\lambda_F(T)\nabla T]=0. \tag{12}$$

Because $\nabla\times\lambda(T)\nabla T=\nabla\times\nabla\left(\int_0^T\lambda(\varepsilon)d\varepsilon\right)$ and $\nabla\times\lambda_F(T)\nabla T=\nabla\times\nabla\left(\int_0^T\lambda_F(\varepsilon)d\varepsilon\right)$, we can obtain $\nabla\times\lambda(T)\nabla T=\nabla\times\lambda_F(T)\nabla T=0$. Substituting it into Eq. (12) leads to

$$(\nabla\times q)_J+\tau\frac{\partial(\nabla\times q)_J}{\partial t}=0 \tag{13}$$

From Eq. (13),

$$(\nabla\times q)_J=(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau}}, \tag{14}$$

which is also an exponential decay function and has the same form as Eq. (11). For the GK model, the rotation [34] is

$$\frac{\partial(\nabla\times q)}{\partial t}+\frac{c^2c_l}{3}\nabla\times\nabla T+\frac{1}{\tau_R}(\nabla\times q)=\frac{3\tau_N c^2}{5}\nabla\times\nabla(\nabla\cdot q). \tag{15}$$

It is not difficult to find that $\nabla\times\nabla T=0$ and $\nabla\times\nabla(\nabla\cdot q)=0$. Therefore, we have

$$\frac{1}{\tau_R}(\nabla\times q)_{GK}+\frac{\partial(\nabla\times q)_{GK}}{\partial t}=0, \tag{16}$$

whose solution is

$$(\nabla\times q)_{GK}=(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau_R}}. \tag{17}$$

From Eq. (17), we can find that the heat flux rotation is also an exponential decay function. In the GK model, there are two kinds of relaxation time, namely τ_N for normal processes and τ_R for momentum loss processes. Eq. (17) shows that the exponential decay of the heat flux rotation is only related to τ_R, which is for dissipative physical processes. Therefore, the heat flux rotation, which is exponentially dissipative, only reflects the dissipative relaxation in the GK model. For the DPL model,

$$(\nabla\times q)_{DPL}+\tau_q\frac{\partial(\nabla\times q)_{DPL}}{\partial t}=0. \tag{18}$$

From Eq. (18),

$$(\nabla\times q)_{DPL}=(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau_q}}. \tag{19}$$

Similarly to other heat conduction models, for the DPL model, the exponential decay of the heat flux rotation is also found in Eq. (19). There are also two kinds of relaxation time in the DPL model, namely τ_q for the heat flux relaxation and τ_T for the temperature gradient relaxation. The exponential decay of the heat flux rotation is only related to τ_q, which is for the heat flux relaxation processes. This means that in the DPL model, the decaying heat flux rotation only reflects the heat flux relaxation.

3. Non-Fourier heat conduction

Without heat source, the heat conduction equation of the CV model is

$$\frac{\partial T}{\partial t}+\tau\frac{\partial^2 T}{\partial t^2}=\frac{\lambda_{CV}}{\rho c_V}\nabla^2 T, \tag{20}$$

which is a hyperbolic equation. The CV model is usually considered to predict wave phenomena in heat conduction processes with finite wave velocity $\sqrt{\frac{\lambda_{CV}}{\rho c_V\tau}}$, but Fourier's law is often considered to predict heat transport in an infinite velocity [35]. Therefore, it is often believed that wave-like heat transport with finite wave velocity is caused by non-Fourier heat conduction models, and this characteristic can be used to make a distinction between Fourier and non-Fourier heat conduction. However, it has been found that Fourier's law can also predict wave-like transport with finite speed in certain heat conduction problems [36]. Especially for non-linear heat source problems [36], it is found that Eq. (3) has travelling wave solution in many cases, such as $\phi=\phi(T)=C_1T^2-C_2T$ and $\lambda_F=C_F$, where C_F, C_1 and C_2 are all constants. Besides non-linear heat source, the non-linear thermal conductivity can also predict heat wave. For example, when the thermal conductivity $\lambda_F(T)$ satisfies $\lambda_F(T) = \lambda_0 T^n$ (λ_0 and n are constants, and $n \neq 0$), Eq. (3) has travelling wave solution as follows

$$T(x,t) = n\left[v\left(x + \frac{v\lambda_0 t}{\rho c_V}\right) + c\right]^{\frac{1}{n}}, \tag{21}$$

where v and c are constants. Eq. (21) shows that the thermal conductivity satisfying $\lambda_F(T) = \lambda_0 T^n$ can lead to diffusion wave with wave velocity $\left|\frac{v\lambda_0}{\rho c_V}\right|$. One of the most well-known mediums satisfying this condition is ideal gas, whose thermal conductivity is proportional to $T^{\frac{1}{2}}$. Therefore, in the above cases, heat wave with finite transport velocity cannot prove that non-Fourier heat conduction occurs. Even for constant physical properties problems, it is possible that heat wave cannot prove non-Fourier heat conduction either.

As typical examples, we make a discussion on Fourier's law and the CV model. For one-dimensional heat conduction without heat source $\phi = 0$, when all physical properties are constants and $\lambda_{CV} > \lambda_F$, consider a travelling wave solution

$$T(x,t) = C_3 \exp\left[\frac{t}{\tau}\left(\frac{\lambda_{CV}}{\lambda_F} - 1\right) \pm x\sqrt{\frac{\rho c_V}{\tau\lambda_F}\left(\frac{\lambda_{CV}}{\lambda_F} - 1\right)}\right], \tag{22}$$

where C_3 is a constant. Travelling wave solution Eq. (22) can satisfy both Eq. (3), which is for Fourier's law, and Eq. (20), which is for the CV model. In this problem, the two models have a same wave velocity of the heat transport $\sqrt{\frac{\lambda_{CV}-\lambda_F}{\rho c_V \tau}}$. However, the CV model is often considered to predict wave velocity as $\sqrt{\frac{\lambda_{CV}}{\rho c_V \tau}}$, which is obviously larger than the wave velocity $\sqrt{\frac{\lambda_{CV}-\lambda_F}{\rho c_V \tau}}$ in this problem. Eq. (22) means that even for constant physical properties problems, both Fourier's law and the CV model can predict exactly the same heat wave, and exactly the same temperature field. Therefore, it could be infeasible to prove that a heat conduction process is non-Fourier from the viewpoint of finite transport velocity or heat wave phenomena. That is because both heat wave phenomena and transport velocity are determined by the solution of the governing equation, and the solution reflects the mutual influence of the governing equation and the conditions determining the solution. The governing equation is derived from the heat conduction law and the energy conservation equation, which reflects the influence of the heat source. The conditions determining the solution is composed of the initial and boundary conditions. Therefore, not only the heat conduction law, but also some other factors, including the heat source, the initial conditions and the boundary conditions, can influence the wave characteristics and transport velocity. Therefore, wave-like heat transport and finite transport velocity cannot reflect the characteristics of non-Fourier heat conduction directly, and they are not the essential distinction between Fourier's law and non-Fourier heat conduction models.

Different from wave characteristics and transport velocity, Section 2 shows that the change rule of the heat flux rotation is only determined by the heat conduction laws. For a heat conduction model, the change rule of the heat flux rotation is established, which is not influenced by the heat source, the initial conditions, or the boundary conditions. Therefore, compared with wave characteristics and transport velocity, the heat flux rotation (vortex) reflects the characteristics of the heat conduction models better by eliminating the influence from other factors. From Section 2, it is found that the heat flux of Fourier heat conduction must be non-rotating, and for typical non-Fourier heat conduction models, including the CV model, Jeffrey model, the GK model and the DPL model, the heat flux rotation must be exponentially dissipative. Because the influence of other factors is eliminated, it is no doubt that the difference for heat flux rotation is only caused by non-Fourier effect. Therefore, the heat flux rotation can be considered as a distinction between Fourier's law and non-Fourier heat conduction models. This conclusion provides a perspective for proving non-Fourier heat conduction, which could prove non-Fourier heat conduction more rigorously than the methods based on heat wave phenomena and finite transport velocity. For proving non-Fourier heat conduction in experiments and simulation data, this perspective based on the heat flux rotation can be applied by observing the heat flux circulation Π_q, which is expressed as

$$\Pi_q = \int_{\partial\Gamma} q \cdot \vec{\tau}\, dl = \int_{\Gamma} (\nabla \times q) \cdot \vec{n}\, dS, \tag{23}$$

where Γ is an orientable surface, $\partial\Gamma$ is the positive boundary, $\vec{\tau}$ is the unit tangent vector of $\partial\Gamma$, $\vec{n}$ is the positive unit normal vector, dl is the line element and dS is the surface element.

If non-zero heat flux circulation is found in experiments or simulation data, we can prove that non-Fourier heat conduction occurs. What's more, we can also check if the heat flux circulation is exponentially dissipative to determine if some non-Fourier heat conduction models, such as the CV model, is satisfied in experiments or simulation data.

The above perspective can also make a distinction between some non-Fourier heat conduction models. For example, for the T-T model [16], which is for heat conduction in metals, the heat flux satisfies the constitutive relation

$$q = -k_e \nabla T_e, \tag{24}$$

where T_e is the electron temperature and k_e is the electron thermal conductivity. It is not difficult to find that $(\nabla \times q)_{T-T} = 0$. Although Eq. (24) is very similar to Fourier's law, the heat conduction equation of the T-T model is very different

$$\nabla^2 T_e + \frac{\alpha_e}{C_E^2}\frac{\partial}{\partial t}(\nabla^2 T_e) = \frac{1}{\alpha_E}\frac{\partial T_e}{\partial t} + \frac{1}{C_E^2}\frac{\partial^2 T_e}{\partial t^2}, \tag{25}$$

where α_E is the equivalent thermal diffusivity, α_e is the thermal diffusivity of the electrons and C_E is the heat wave velocity. Eq. (25) is quite different from the heat conduction equation of Fourier's law, but has the same form as the heat conduction equation of the GK model, which is expressed as

$$\nabla^2 T + \frac{9\tau_N}{5}\frac{\partial}{\partial t}(\nabla^2 T) = \frac{2}{\tau_R c^2}\frac{\partial T}{\partial t} + \frac{3}{c^2}\frac{\partial^2 T}{\partial t^2}. \tag{26}$$

Both Eq. (25) and Eq. (26) contain differential terms $\frac{\partial}{\partial t}$, $\frac{\partial^2}{\partial t^2}$, ∇^2 and $\frac{\partial}{\partial t}(\nabla^2)$. When the coefficients of these differential terms are proportional

$$\frac{5\alpha_e}{9\tau_N C_E^2} = \frac{\tau_R c^2}{2\alpha_E} = \frac{c^2}{3C_E^2}, \tag{27}$$

The GK model and the T-T model will lead to exactly the same temperature field for the same initial and boundary conditions. Therefore, we cannot distinguish the two models from the characteristics of temperature field when the coefficients are proportional. However, their characteristics of the heat flux rotation are completely different. The heat flux rotation of the GK model is exponentially dissipative, but the T-T model must lead to non-rotating heat flux. Therefore, the heat flux rotation can be considered as a perspective for distinguishing the two models.

4. Physical discussion

The heat flux rotation shows the rotating motion of heat flux

$$\nabla \times q = 2\varpi_q, \tag{28}$$

where ϖ_q is the angular velocity of heat flux field. Therefore, the conclusions in Section 3 can also be established between the angular velocity of heat flux and heat conduction laws. Then, connections between physical phenomenon and physical mechanisms in heat conduction processes can be built as follows. The angular velocity or angular displacement of heat flux must be caused by non-Fourier heat conduction mechanisms. The decay rate of the angular velocity reflects certain relaxation processes determined by heat conduction mechanisms. For Fourier heat conduction, whose angular velocity of heat flux must be zero, the decay rate of the angular velocity could be understood as $+\infty$, because the relaxation process doesn't exist.

Heat conduction equation of the CV model, Eq. (20), has the same form as the wave equations in mechanical phenomenon. Because of this similarity of the governing equations, non-Fourier heat conduction is considered to have similar characteristics belonging to mechanical motion, such as "heat inertia" [37]. However, the rotating motion of heat flux is only determined by physical constitutive relation, which is quite different from the rotating motion in mechanics. For example, in Newtonian fluid, the rotation Ω of velocity field satisfies

$$\frac{\partial\Omega}{\partial t}+\nabla\times(\Omega\times V)=\nabla\times F-\nabla\times\left(\frac{\nabla p}{\rho}\right)+(\Omega\cdot\nabla)V-\Omega(\nabla\cdot V)+\nu\nabla^2\Omega, \tag{29}$$

where V is the velocity field, $\Omega=\nabla\times V$, F is the body force, p is the pressure and ν is the kinematic viscosity. From Eq. (29), obviously, the rotating motion in Newtonian fluid cannot be determined by Newton's law of viscosity, because the body force will also have an effect, which can have nothing to do with the constitutive relation. What's more, Eq. (29) also contains V and p, which reflect the influence of the initial and boundary conditions. In mechanics, inertia means that changing the motion states, including the rotating motion, needs the action of forces. In other words, the action of forces can change the motion states, and makes the rotating motion to be increasing, decaying or constant. However, for a non-Fourier heat conduction model, the exponential decay of the rotating motion is already established, which seems an "infinite inertia" for rotating motion. Therefore, although the governing equations of non-Fourier heat conduction are similar to the governing equations in mechanical phenomenon, the rotating motion of heat flux shows particular characteristics of non-Fourier heat conduction.

5. Entropy flux rotation

Besides heat flux, entropy flux $J^S=\frac{q}{T}$ is another common and important flux in heat conduction processes, whose rotation is calculated as

$$\nabla\times J^S=\nabla\times\left(\frac{q}{T}\right)=\frac{\nabla\times q}{T}-\frac{\nabla T\times q}{T^2}. \tag{30}$$

For Fourier's law, we have $\nabla\times q=0$ and therefore, the entropy flux rotation is

$$\nabla\times J^S_F=\left[\nabla\times\left(\frac{q}{T}\right)\right]_F=0+\frac{\lambda_F\nabla T\times\nabla T}{T^2}=0, \tag{31}$$

which shows that entropy flux of Fourier's law is also irrotational. For the CV model, substituting Eqs. (4) and (11) into Eq. (30) leads to

$$\begin{aligned}\nabla\times J^S_{CV}&=\frac{(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau}}}{T}+\frac{(\tau\frac{\partial q}{\partial t}+q)\times q}{\lambda_{CV}T^2}\\&=\frac{(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau}}}{T}+\frac{\tau\frac{\partial q}{\partial t}\times q}{\lambda_{CV}T^2}.\end{aligned} \tag{32}$$

For Jeffrey model, substituting Eqs. (5) and (14) into Eq. (30) leads to

$$\nabla\times J^S_J=\frac{(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau}}}{T}+\frac{[\tau\frac{\partial q}{\partial t}+\lambda_F\frac{\partial(\nabla T)}{\partial t}]\times q}{kT^2}. \tag{33}$$

For the GK model, the entropy flux rotation is

$$\nabla\times J^S_{GK}=\frac{(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau_R}}}{T}+\frac{3\left[\frac{\partial q}{\partial t}-\frac{3\tau_N c^2}{5}\nabla(\nabla\cdot q)\right]\times q}{c^2c_lT^2}. \tag{34}$$

For the DPL model, the entropy flux rotation is

$$\nabla\times J^S_{DPL}=\frac{(\nabla\times q)|_{t=0}e^{-\frac{t}{\tau_q}}}{T}+\frac{\left[\tau_q\frac{\partial q}{\partial t}+\lambda_{DPL}\tau_T\frac{\partial(\nabla T)}{\partial t}\right]\times q}{\lambda_{DPL}T^2}. \tag{35}$$

It is not difficult to find that different from the heat flux rotation, the entropy flux rotation of a non-Fourier heat conduction model does not change in an established rule determined by the model. The heat flux field and temperature field also have influence on the entropy flux rotation, which shows that the entropy flux rotation is also related to the heat source, the initial conditions, and the boundary conditions.

6. Conclusions

1. The heat flux rotation of Fourier's law must be non-rotating, and for several typical non-Fourier heat conduction models, including the CV model, Jeffrey model, GK model and DPL model, the rotation is exponentially dissipative. The decay rate of the heat flux rotation can reflect certain relaxation processes in non-Fourier heat conduction.
2. Essentially speaking, heat wave phenomena and finite transport velocity reflect the mutual influence of the governing equation and the conditions determining the solution, rather than heat conduction law, which is only one of the factors for deriving the governing equation. As comparison, the change rule of the heat flux rotation is only determined by the heat conduction law. Therefore, the heat flux rotation could provide a perspective for proving non-Fourier heat conduction, which is more rigorous than heat wave phenomena or finite transport velocity. In experiments and simulation data, this perspective can be applied by observing the heat flux circulation.
3. The heat flux rotation shows the rotating motion of heat flow, which reflects certain relaxation processes in heat conduction. The characteristics of the heat flux rotating motion, which is exponentially dissipative, are quite different from the characteristics of the rotating motion in fluid mechanics. This shows that although the governing equations of non-Fourier heat conduction are similar to the governing equations in mechanical phenomenon, non-Fourier heat conduction laws have particular characteristics for the rotating motion.
4. The entropy flux of Fourier's law is still irrotational. Different from the heat flux rotation, the entropy flux rotation of a non-Fourier heat conduction model does not change in an established rule determined by the model.

Conflicts of interest

The author declares no conflict of interest.

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